

# EVOLUTIONARY TIME SCALES FOR LATE CARBON-BURNING PHASES WITH NEUTRINO EMISSION

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## ABSTRACT

The effect of the two most important neutrino processes (pair annihilation and photoneutrino) is evaluated for stars during the carbon-burning stage. At this point,  $L_\nu > L$  (photon), and the evolution of the core can be considered separate from that of the envelope. The time scale of evolution from carbon depletion to the onset of neon-burning is estimated from representative central densities and the corresponding critical temperatures,  $T \gtrsim 0.5 \text{ B}^\circ \text{ K}$ , at which the energy generation due to the  $\text{C}^{12} + \text{C}^{12}$  reaction is equaled by the energy loss from neutrinos. If the degeneracy is high ( $\rho \sim 10^6 \text{ gm/cm}^3$ ), neutrino emission accelerates cooling of the star. In both cases the evolution is limited only by the rate of neutrino emission.

## I. INTRODUCTION

Stellar evolutionary sequences have been integrated, with more or less success on the basis of current theory, through the helium-burning stage. Hayashi and Cameron (1962) have, in fact, considered preliminary models of helium depletion and the onset of carbon-burning for a star of  $15.6 M_\odot$ . However, their work does not consider neutrino emission, which should become important during carbon-burning (see Sec. II). Depending on the stellar mass (and hence the degree of degeneracy of the gas), neutrino emission may serve either to cool off the star or to accelerate the evolution by a gravitational contraction to the point of more advanced nuclear reactions. In the light of qualitative research on the evolution of massive stars into presupernovae (Chiu 1961*a, b*; Chiu and Fuller 1962; Reeves 1962*b*), it is of interest to calculate the time scales of evolution for a variety of stellar masses undergoing dominant neutrino emission when carbon-burning is on the wane.

## II. NUCLEAR AND NEUTRINO PROCESSES

Current theory predicts that when  $T \gtrsim 0.5 \text{ B}^\circ \text{ K}$  (billion degrees), a star will be in the carbon-burning stage (Reeves and Salpeter 1959; Reeves 1962*b*). In this paper we consider the heavy-ion carbon reaction  $\text{C}^{12} + \text{C}^{12}$ , relevant to the commencement of carbon-burning; the most abundant isotopes produced are  $\text{Ne}^{20}$ ,  $\text{Na}^{23}$ , and  $\text{Mg}^{24}$  (Reeves and Salpeter 1959). Recently the cross-section for this reaction has been experimentally determined at Chalk River and the energy generation calculated by Reeves (1962*a*).

Actually, for stars in the mass range of interest in this paper,  $0.4 \lesssim M/M_\odot \lesssim 10$  (see Sec. III), the abundance ratio of  $\text{C}^{12}$  to  $\text{O}^{16}$  produced in the helium-burning stage will be about 3; the abundance of neon is negligible (Reeves 1962*b*). However, the  $\text{C}^{12} + \text{O}^{16}$  reaction is not expected to be astrophysically important (Reeves 1962*a*), and our assumption of a pure carbon core is allowable.

The two most important modes of neutrino creation inside a star have been shown to be pair annihilation (Chiu and Morrison 1960; Chiu 1961*c*),

$$e^- + e^+ \rightarrow \nu + \bar{\nu}, \quad (1)$$

and the photoneutrino process (Chiu and Stabler 1961; Ritus 1961),

$$\gamma + e^- \rightarrow e^- + \nu + \bar{\nu}, \quad (2)$$

among all neutrino processes discussed (see Chiu and Stabler 1961). Since the great increase in neutrino energy density at  $T \sim 0.5$  B° K causes the neutrino luminosity to rise above the photon luminosity, the cooling time decreases considerably (see, e.g., Chiu 1961*a*; Stothers and Chiu 1962). The evolution of the star is accelerated because stellar structure must comply with the virial theorem. Only if the energy generated by nuclear reactions is comparable with the neutrino energy loss can the gravitational contraction be halted.

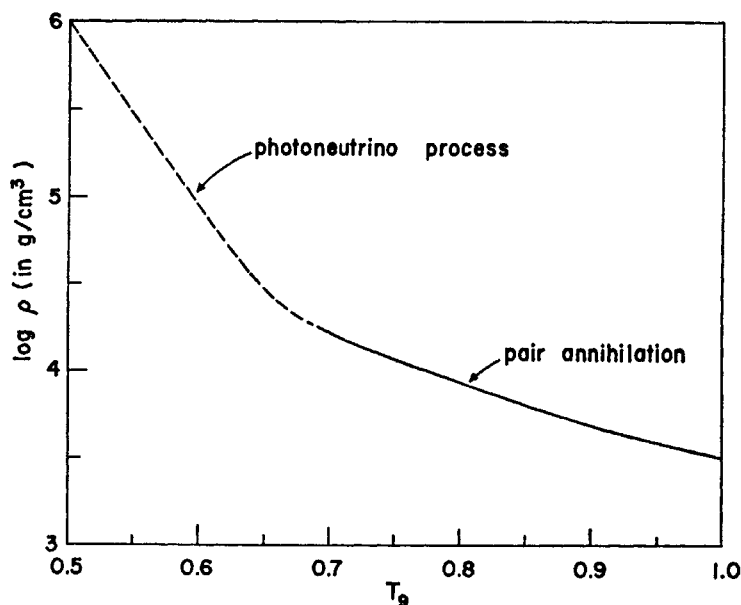


FIG. 1—Curve represents  $\epsilon_{\text{nuc}} = \epsilon_{\nu}$ , where  $\epsilon_{\text{nuc}}$  is due to the  $\text{C}^{12} + \text{C}^{12}$  reaction and  $\epsilon_{\nu}$  is due to pair annihilation and photoneutrinos.

TABLE 1  
STELLAR MODELS FOR  $\epsilon_{\text{nuc}} = \epsilon_{\nu}$

$\log \rho$ ( $\text{gm}/\text{cm}^3$ )	$T_{\text{crit}}$ (B° K)	Equation of State	$M(M_{\odot})$
6	0.5	Degenerate electrons	0.4
5	6	Gas	1.4
4	0.8	Gas and radiation	9

In Figure 1 we have plotted a line on the  $\log T - \log \rho$  plane along which the two energy rates are equal. To the right of this line lies the region of stellar stability, since here the energy generation due to the  $\text{C}^{12} + \text{C}^{12}$  reaction is greater than the neutrino energy losses. We have chosen three representative densities and listed the critical temperatures in Table 1; the relevant form of the equation of state is obtained by visual inspection of the  $\log T - \log \rho$  diagram found in Schwarzschild (1958, p. 54). The masses are obtained from polytropic models, as indicated in the following section.

### III. MODEL STARS

In this section we shall consider situations that may be encountered by stars evolved to the carbon-burning stage. The evolution of the core is the important factor for the

star at this point (Chiu 1961*a, b*), and we shall be guided by Table 1 and the usual equations of stellar structure (see, e.g., Schwarzschild 1958).

The energy generation will be given by

$$\frac{dL_r}{dM_r} = \epsilon_{\text{nuc}} + \epsilon_{\text{grav}} - \epsilon_\nu = \epsilon_{\text{nuc}} - P \frac{dv}{dt} - \frac{du_{\text{th}}}{dt} - \epsilon_\nu, \quad (3)$$

where  $v = 1/\rho$  is the specific volume of a gram of matter and  $u_{\text{th}}$  is the thermal specific energy. Now observations have shown that  $L$  (photon)  $< 10^{4.5} L_\odot$  for stable stars. Therefore, in the mass range  $M \geq 0.4 M_\odot$  of Table 1,  $dL_r/dM_r \approx L/M < 10^5$ . But at the lowest  $T_{\text{crit}} (= 0.5 \text{ B}^\circ \text{K})$ ,  $\epsilon_\nu > 10^5$ , so that we may always neglect  $dL_r/dM_r$  in comparison with  $\epsilon_\nu$ . Hence  $L_r$  (photon) can be ignored, compared with  $L_\nu$  (see Chiu 1961*b*). We also neglect the addition of mass to the core by nuclear burning in surrounding shells.

$$a) \text{ Case } \log \rho = 6, \epsilon_{\text{nuc}} = \epsilon_\nu \approx 0$$

In this case the core is degenerate and, from the theory of degenerate configurations, will have a mass of  $0.4 M_\odot$  (Chandrasekhar 1939). Nuclear burning has ceased, and we neglect the effect of residual neutrino emission, since the temperature of the isothermal core of ordinary white dwarfs is too low to make this quantity important. Thus further gravitational contraction is negligible, and the sole energy source remaining is the thermal energy of the non-degenerate nuclei. We have

$$\frac{dL_r}{dM_r} = -\frac{d}{dt} \left( \frac{3}{2} \Re T \right), \quad (4)$$

where  $\Re = k/\mu H$  is the gas constant. Thus

$$L = -\frac{d}{dt} \left( \frac{3}{2} \Re T M \right). \quad (5)$$

For a white dwarf (Schwarzschild 1958, p. 237),

$$L = K(\mu, M) T^{3.5}. \quad (6)$$

Therefore,

$$\frac{dT}{dt} = CT^n, \quad \text{where} \quad C = -\frac{2}{3} \frac{K}{\Re M}. \quad (7)$$

This is a degenerate form of Bernoulli's equation of index  $n = 3.5$ . Integration readily yields the cooling time from "infinite" temperature,

$$\tau = \frac{3}{2} \frac{\Re M T}{2.5 L}. \quad (8)$$

For a given mass, luminosity, and chemical composition we may compute the temperature of the isothermal core from equation (6). Then the cooling time follows immediately from equation (8). The cooling times of a pure carbon core ( $\mu_A = 12$  and assume  $[t/\bar{g}] = 10$ ) are listed in Table 2 for various luminosities and the corresponding temperatures (in millions of degrees). These may be compared with cooling times derived for predominantly helium cores ( $\mu_A = 4.44$ ,  $Z = 0.1$ , and  $[t/\bar{g}] = 10$ ) of the same mass and luminosity. Since  $\tau$  depends on the chemical composition in the following way:

$$\tau \propto \frac{1}{\mu_A} \left[ \frac{Z(1+X)\mu_B^2}{(t/\bar{g})\mu} \right]^{1/3.5}, \quad (9)$$

we can calculate the ratio  $\tau(\text{C})/\tau(\text{He})$  to be 0.7.

b) Case  $\log \rho = 6$ ,  $\epsilon_{\text{nuc}} \ll \epsilon_\nu$

We consider the core effectively devoid of nuclear fuel but the temperature sufficiently high to make neutrino emission important. For  $\epsilon_{\text{nuc}} \ll \epsilon_\nu$ ,  $T < T_{\text{crit}} = 0.5 \text{ B}^\circ \text{K}$ , so that the core may be degenerate enough to resist much further gravitational contraction. Then

$$\epsilon_\nu = -\frac{d}{dt}\left(\frac{3}{2}\mathcal{R}T\right). \quad (10)$$

From Figure 1 the photoneutrino process is dominant; therefore, let  $\epsilon_\nu = \epsilon_{\nu 0} T^n$ . We again obtain Bernoulli's equation of index  $n$  with  $C = -\frac{2}{3} \epsilon_{\nu 0} / \mathcal{R}$ . The solution is

$$\tau = \frac{3}{2} \frac{\mathcal{R}T}{(n-1)\epsilon_\nu}. \quad (11)$$

TABLE 2  
COOLING TIMES FOR WHITE DWARFS WITH CARBON CORES ( $0.4 M_\odot$ )

$T$ ( $\text{M}^\circ \text{K}$ )	$\tau$ (years)	$L$ ( $L_\odot$ )	$T$ ( $\text{M}^\circ \text{K}$ )	$\tau$ (years)	$L$ ( $L_\odot$ )
11	$3 \times 10^9$	$10^{-4}$	40	$0.1 \times 10^9$	$10^{-2}$
20	$0.6 \times 10^9$	$10^{-3}$			

TABLE 3  
COOLING TIMES FOR HOT DEGENERATE CARBON CORES  
WITH NEUTRINO EMISSION ( $0.4 M_\odot$ )

$T$ ( $\text{B}^\circ \text{K}$ )	$\tau$ (years)	$L_\nu$ ( $L_\odot$ )	$T$ ( $\text{B}^\circ \text{K}$ )	$\tau$ (years)	$L_\nu$ ( $L_\odot$ )
0.2	$7 \times 10^4$	70	0.4	$6 \times 10^2$	$2 \times 10^4$
0.3	$4 \times 10^3$	$2 \times 10^3$	0.5	$1 \times 10^2$	$1 \times 10^5$

We note that, apart from the numerical factor  $n - 1$ , this is simply the thermal energy divided by the neutrino loss rate.

The rate of photoneutrino energy loss goes as  $T^8$  in the case of non-degenerate electrons. The degenerate case has not yet been satisfactorily worked out (Reeves 1962*b*), but the reduction of phase space available to the outgoing electron will lower the rate. Thus the cooling times listed in Table 3 are lower limits. However, in the temperature range of Table 3, the degeneracy is only partial, and the assumption of the non-degenerate photoneutrino rate will not be badly off, especially since it may even rise slightly during partial degeneracy. On the other hand, we have neglected  $\epsilon_{\text{grav}}$ , and the appropriate mass will be greater than  $0.4 M_\odot$ . It will not be much greater because a perfect gas model (polytrope of index 1.5) would predict  $\sim 0.4 M_\odot$  for the density and temperature considered here. Nevertheless, we can obtain a rough idea of the neutrino luminosity by assuming a uniform energy source in an isothermal core and  $M = 0.4 M_\odot$ . In Table 3 we have listed  $L_\nu$  for various temperatures.

If the temperature drops much lower, the photon luminosity will have to be taken into account.

c) Case  $\log \rho = 5$ ,  $\epsilon_{\text{nuc}} \ll \epsilon_\nu$

In this non-degenerate case, we must allow fully for the contraction of the core. We have, from equation (3),

$$\epsilon_\nu = -P_g \frac{dv}{dt} - \frac{d}{dt} \left( \frac{3}{2} \Re T \right). \quad (12)$$

Again let  $\epsilon_\nu = \epsilon_{\nu 0} T^n$  for the photoneutrino process and assume homologous contraction of our perfect-gas sphere. Then the evolution follows from Lane's law (see, e.g., Chandrasekhar 1939) and is represented by

$$x(t) = x(t_0), \quad (13)$$

where  $x = T^3/\rho$ . Since the energy-balance equation is taken at any mass element  $M_r$ , we can consider the evolution of the center. Then equation (12) reduces to equation (7) with  $C = \frac{2}{3} \epsilon_{\nu 0}/\Re$ . Thus

$$\tau = \frac{3}{2} \frac{\Re}{n-1} \left[ \left( \frac{T}{\epsilon_\nu} \right)_0 - \frac{T}{\epsilon_\nu} \right], \quad (14)$$

where we set  $T = T_0$  at the initial time from which  $\tau$  is computed.

TABLE 4  
CONTRACTION TIMES FOR EXHAUSTED CARBON CORE WITH DOMINANT GAS PRESSURE AND NEUTRINO EMISSION ( $1.6 M_\odot$ )

$T_0$ (B° K)	$\log \rho_c$ (gm/cm <sup>3</sup> )	$\tau$ (years)	$L_\nu$ ( $L_\odot$ )
0.67	5.00	0	$1 \times 10^5$
0.70	5.06	28	$2 \times 10^5$
0.75	5.15	59	$4 \times 10^5$
0.8	5.23	77	$7 \times 10^5$
1.0	5.52	101	$7 \times 10^6$
1.2	5.76	106	$5 \times 10^7$
1.4	5.96	107	$2 \times 10^8$

In Table 4 we have arbitrarily chosen  $T_0$  as the central temperature for which  $L_{\text{nuc}} = L_\nu$  in a polytrope of index 1.5 (perfect gas), assuming a pure carbon core. Since the neutrino loss extends over a greater volume of the star than does the carbon-burning rate,  $T_0$  is slightly greater than  $T_{\text{crit}}$  for  $\epsilon_{\text{nuc}} = \epsilon_\nu$  in Table 1. The polytropic mass is  $1.6 M_\odot$  and remains constant for a homologous contraction.

d) Case  $\log \rho = 4$ ,  $\epsilon_{\text{nuc}} \ll \epsilon_\nu$

Here we must add the contribution from radiation pressure to equation (12):

$$\epsilon_\nu = -(P_g + P_r) \frac{dv}{dt} - \frac{d}{dt} \left( \frac{3}{2} \Re T + \frac{aT^4}{\rho} \right). \quad (15)$$

From Table 1 and Figure 1 we see that pair annihilation is the dominant neutrino process; therefore, let  $\epsilon_\nu = \epsilon_{\nu 0} T^n/\rho$ . Now the evolution of a perfect-gas sphere with radiation included can easily be shown to be the same as equation (13). If we consider only the center, equation (15) yields

$$\frac{dT}{dt} - \left( \frac{8}{9} \frac{ax}{\Re} + \frac{2}{3} \right) \frac{T}{x} \frac{dx}{dt} = CT^{n-3}, \quad (16)$$

where  $C = \frac{2}{3} \epsilon_{\nu 0} x / \mathfrak{R}$ . Since  $x$  remains constant in time, however, this equation reduces to Bernoulli's equation of index  $n - 3$ . We easily obtain

$$\tau = \frac{3}{2} \frac{\mathfrak{R}}{n - 4} \left[ \left( \frac{T}{\epsilon_{\nu}} \right)_0 - \frac{T}{\epsilon_{\nu}} \right]. \quad (17)$$

This is evidently the same as equation (14) for the photoneutrino process and gas pressure alone, excepting the numerical factor  $n - 4$ .

Table 5 contains the contraction times of a polytrope of index 3 (gas plus radiation) and mass  $10 M_{\odot}$  in accordance with an initial  $T_0$  computed from  $L_{\text{nuc}} = L_{\nu}$ .

TABLE 5  
CONTRACTION TIMES FOR EXHAUSTED CARBON CORE WITH GAS PLUS RADIATION PRESSURE AND NEUTRINO EMISSION ( $10 M_{\odot}$ )

$T_c$ (B° K)	$\log \rho_c$ (gm/cm <sup>3</sup> )	$\tau$ (days)	$L_{\nu}$ ( $L_{\odot}$ )
0 82	4 00	0	$3 \times 10^8$
0 85	4 05	149	$5 \times 10^8$
0 90	4 12	275	$1 \times 10^9$
0 95	4 19	331	$2 \times 10^9$
1 0	4 26	359	$5 \times 10^9$
1 2	4 50	388	$4 \times 10^{10}$
1 4	4 70	393	$2 \times 10^{11}$

#### IV. DISCUSSION

When the temperature reaches  $1.3 - 1.5 \text{ B}^{\circ} \text{K}$ , neon-burning will have commenced in the carbon-depleted core (Reeves 1962*b*). These temperatures are those at which  $\epsilon_{\text{nuc}}(\text{neon}) = \epsilon_{\nu}$  in the mass range of interest in this paper. Thus the contraction time at  $T \sim 1.4 \text{ B}^{\circ} \text{K}$  in Tables 4 and 5 is the time spent between the onset of gravitational contraction during carbon exhaustion and the onset of neon-burning. The delaying effect of increased carbon-burning after a gravitationally induced temperature rise is minor because not only is the core already comprised mostly of neon, which cannot yet react, but also the remaining carbon is consumed at an increasingly faster rate. However, since carbon-burning is expected to be dominant in the temperature range  $0.6 < T < 0.9 \text{ B}^{\circ} \text{K}$  (Reeves and Salpeter 1959) and we assumed a pure carbon density at  $T_0$ , the contraction times for  $T \lesssim 0.8 \text{ B}^{\circ} \text{K}$  in Table 4 and  $T \lesssim 1.0 \text{ B}^{\circ} \text{K}$  in Table 5 may be too small. The choice of initial  $T_0$  is actually unimportant, however, because the contraction time between two temperatures, as measured by the difference of the two corresponding  $\tau$ , is independent of  $T_0$ .

In this paper we have assumed that the core evolves homologously. In fact, that  $T_c^3/\rho_c = \text{constant}$  is not a bad assumption for non-degenerate cores during gravitational contraction phases is substantiated by detailed model calculations (e.g., Schwarzschild 1958; Hayashi and Cameron 1962).

To summarize: once gravitational contraction begins because of instability induced when the nuclear energy generation is insufficient to supply the stellar energy requirements ( $\epsilon_{\text{nuc}} < \epsilon_{\nu}$ ), homologously contracting models show that the time scale of evolution (eqs. [14] and [17]) is governed strictly by the rate of neutrino emission.

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